A risk-oriented degradation model for maintenance of reinforced concrete structure subjected to cracking

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Abstract

This article is within the context of decision models aimed for maintenance of structures and infrastructures in civil engineering. The contribution relies on the construction of a degradation model oriented toward risk analysis. The proposed model can be defined as a meta-model in the sense that it is based on observations while incorporating key features from the degradation process necessary for the maintenance decision. We propose to stimulate the construction of the degradation model based on the crack propagation of a submerged reinforced concrete structure subject to chloride-induced corrosion. Furthermore, a set of numerical illustrations is performed to demonstrate the advantages and applicability of the proposed approach in risk management and maintenance contexts.

Keywords

Decision models, maintenance, civil engineering, degradation model, meta-model, chloride-induced corrosion, risk analysis, risk management

Introduction

Investments in maintenance optimization of structures and infrastructures continue to grow due to many reasons such as the reduction in resources allocated to maintenance, the increasingly aging structures, and the firming up of security requirements linked to increased social constraints. Within this context, we can find diverse, yet complementary research axes that need to be developed.

Before going further on some of these axes, we take one step back to present the requirements of modeling and optimizing of risk-based maintenance. A risk-based maintenance model can be defined as the relationship between reliability and performance model or a system's degradation model, the available information that can be used to assess the condition of the structure, a set of decisions and actions, and finally a decision-making framework formed from a set of objectives and requirements. The objective of a maintenance model is then to provide a set of guidelines for the maintenance activities on the basis of current data of the structure with the aim of optimizing decision criteria that sometimes can be complex even contradictory.

Now, we proceed to classify some of the different research axes in the field of maintenance in civil engineering according to their priority. We can start by highlighting the efforts invested in research for the construction of degradation models. These studies deepen our knowledge in analyzing and understanding civil engineering pathologies (e.g. to help predict remaining useful lifetime[1]). In a classical mechanical approach, all available physical knowledge is introduced in the degradation model, leading to the production of the so-called physics-based models. These models can be used to estimate the effects of loads and constraints as well as the spread of damage within a structure at a microscopic level. However, they require

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a thorough knowledge of the constitution of the structure as well as the environment in which it operates.

Furthermore, the literature is rich with research devoted to the development of inspection and monitoring techniques of structures.[2] The challenges in this research axis can be summarized in the identification of indicators of degradation that can adequately account for the process of degradation of the structure. We dispose of a wide range of techniques classed into two categories: non-destructive testing (NDT) which can be used for in-situ tests during inspections, and destructive testing (DT) which are used for testing samples in the laboratory (a controlled environment) to obtain material properties and other detailed information on the condition of the structure and respective deterioration processes. A technical guide on inspection techniques can be found in the following link: http://durati.lnec.pt. Also, we can emphasize on the difficulty encountered in some cases between measuring the degradation and the degradation mechanisms. In some cases, the degradation is not visible or measurable using NDT, for example, fatigue-motivated degradation (roads, etc.). Fatigue is an internal deterioration mechanism; therefore, we find difficulty in measuring the degradation.

Generally, we can consider that we dispose of a great richness in degradation models, in numerous measuring and inspecting technologies, and in the maintenance and decision-making procedures. However, the combination of all these concepts and their integration in an management scheme remains the hardest and crucial task for any risk management and maintenance optimization procedure. It is within this context that our proposal lays.

With this being said, it is expected from a model of degradation the following properties:

- The ability to model the degradation process;
- The ability to give a prognostic of the performance of the structure;
- The ability to be connected to and updated from available data (NDT);
- The ability to be integrated in complex decision optimization criteria;

Physics-based degradation models were intensively developed in the last half century.[3] They started incorporating more physic-chemical and mechanical couplings. As a result, the numbers of parameters calibrating the models have increased greatly. Also, these approaches face several challenges in a reliability context, especially in the randomization of the model.

On the other hand, we have probabilistic models such as random variables and Markov chains,[4] but these approaches suffer from lack of acceptability by the civil engineering community due to several reasons: lack of data for the calibration, poor parameter identification, restrictive assumptions (especially when the degradation shows non-stationary characteristics), and lack in the application guidelines.

One approach seems promising for maintenance optimization in civil engineering is the construction of data-driven degradation meta-models based on stochastic processes such as the gamma process.[5,6] It allows modeling the evolution of the degradation using observations via NDT while maintaining the most critical aspects of the degradation mechanism in the model for the decision and an ease of integration in a more complex maintenance decision criteria. We may further highlight modeling difficulties when the selected pathologies have non-stationary behavior over time (acceleration or deceleration effects of degradation). We can find extensions called conditional or state-dependent models are used to model these non-stationary effects based solely on levels of degradation.[7,8] However, it may be noted that in the construction of these approaches, the authors failed to find a robust procedure for the identification of input parameters as well as a lack of application procedure, limitations making them difficult to appropriate and validate in an operating context.[9]

The aim of this work is to investigate the ability of the degradation meta-model to respond to common concerns in civil engineering and maintenance optimization, such as heterogeneity in collected field data, risk management applications, and maintenance action modeling.

This work is an extension of the preliminary work presented in El Hajj et al.,[10] where databases were considered perfect (in terms of inspection quality). This work is a part of the SI3M project (2012–2016 Identification of Meta-Model for Maintenance Strategies) funded by *Region Pays de la Loire* (France).

The remainder of this article is organized as follows. Section "Construction of the meta-model of the reinforced concrete structure cracking" explains the corrosion-induced cracking and the construction of the degradation the meta-model associated to it. Section "Statistical performance analyses" investigates the statistical performances of the proposed degradation meta-model when faced with heterogeneity in databases. In section "Application to risk analysis," a risk analysis is proposed to illustrate the potential benefit of the proposed degradation model for risk management. Section "Maintenance action modeling" illustrates how a maintenance action's effect can be integrated in the degradation meta-model. Finally, conclusions and perspectives are drawn in section "Conclusion."

Construction of the meta-model of the reinforced concrete structure cracking

In this section, we present the degradation pathology and its main parameters of interest showing their evolution through time. Then, we develop the meta-model to evaluate these parameters.

Description of the chloride-induced corrosion

Corrosion is the main threat to reinforced concrete (RC) structures.[11,12] If left untreated, corrosion can lead the structure to failure due to spalling or delamination. The role of reinforcements in RC is to strengthen the material property, but can also cause failure because of its own corrosion.

The cracking process of a RC structure can be divided into three phases:[13]

- *Diffusion phase*: characterized by the diffusion of chlorides into the concrete. When the concentration of chloride exceeds a threshold, we have depassivation of the steel and the corrosion will be initiated;
- *Corrosion phase*: dominated by the expansion of corrosion products resulting from the corrosion process. The rust slowly fills the surrounding pores and starts to generate internal tensile stress on the concrete. When the first crack appears, this phase is considered to end;
- *Crack propagation phase*: characterized by the excessive accumulation of rust from the ongoing corrosion process resulting in crack propagation until reaching the ultimate point of rupture.

Selection of the degradation indicators

In this study, we are interested in the crack propagation phase. Within this phase, it was found that the parameters of importance, sufficient for modeling the process at every stage, are the corrosion current density and the width of the crack.[13] In this study, we consider that propagation is resulting directly and solely from corrosion and not fatigue.

The corrosion current density i_{corr} is an instantaneous rate of corrosion measured using NDT, expressed in ($\mu A/cm^2$). It can be used to calculate the corrosion rate V_{corr}(mm year^{-1}) through Faraday's law. The acquiring of i_{corr} is highly sensitive to external conditions (e.g. temperature and humidity), thus, in this context, we should usually dispose of modeling and decision support calibration curves or we must always conduct the inspections under identical conditions. In this study, we consider the second situation; the first one has not been established generally.[14] The crack width can be measured using gauge blocks or image analysis.

Figure 1(a)[15] represents the variation in the corrosion rate on all three phases of the cracking, and Figure 1(b)[16] draws the shape of the variation in the width of a crack versus time for two cases of corrosion rate (invariant or time-varying).

From Figure 1(b), we can see the importance of modeling the corrosion rate; the hypothesis of an invariant corrosion rate is not conservative since it leads to an overestimation of by 100% (from 0.5 to 1 mm in 20 years).

As far as we know, the mutual dependencies between the two degradation indicators of importance of this phase (corrosion current density and crack width) have not been studied. One main reason can be attributed to the need of a high number of experiments that is not available for this particular phase; another good reason is that it is virtually impossible to integrate the mutual dependencies in the available physics-based models. In the present work, we aim to tackle this issue with a focus on risk and reliability assessment.

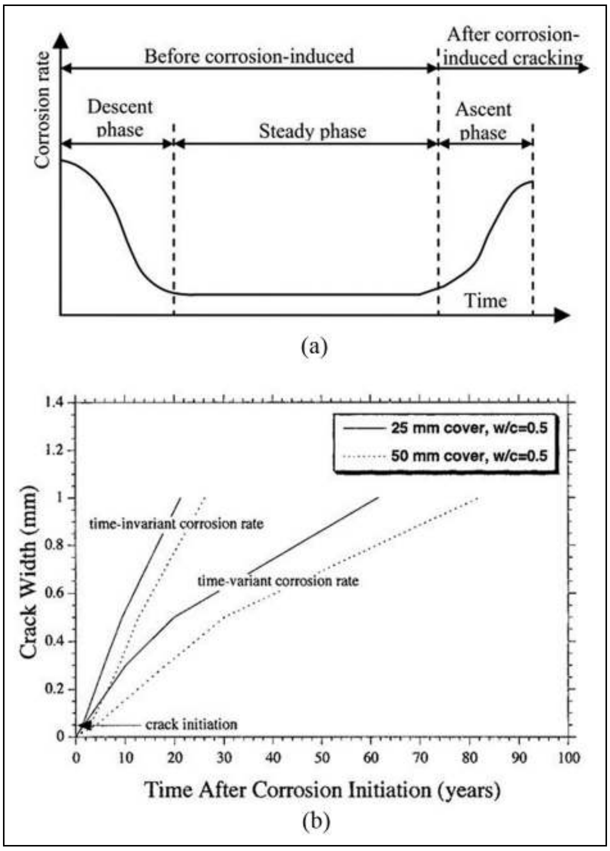

Figure 1. (a) Mean tendency of corrosion rate[15] and (b) Mean tendency of crack width for the third phase.[16]

Construction of the model

The idea of modeling the degradation using two processes can be very rewarding in terms of degradation modeling and maintenance management. Having two sources of inspections instead of one is shown to be more reliable in a maintenance context. Also, this can be appealing to inspection optimization in a sense that the decision maker can combine information coming from two inspection techniques to get a more robust assessment of the condition state.

We propose to define the bivariate process $(\rho_t, \theta_t)_{\forall t \geqslant 0}$ as follows

- $(\rho_t)_{\forall t \geqslant 0}$ modeling the width of the crack (maximum size of one structural component) «*a*» (mm);
- $(\theta_t)_{\forall t \geqslant 0}$ modeling the corrosion current density $<< i_{corr} >>$ ($\mu A/cm^2$)

The two processes $(\rho_t)_{\forall t \geqslant 0}$ and $(\theta_t)_{\forall t \geqslant 0}$, hereafter written ρ and θ, are both dependent and observable.

The evolution of degradation over a period of time τ is given by positive increments for the degradation processes respectively $(\Delta\rho, \Delta\theta)$ which are continuous random variables. Let $g_\tau(x, y; \rho, \theta)$ be the conditional probability density function of $(\Delta\rho, \Delta\theta)$ over the next time period τ given the current degradation level (ρ, θ). The construction of the state-dependent function $g_\tau(x, y; \rho, \theta)$ is based on the process proposed in Zouch et al.[8] and developed in El Hajj et al.[10] Both of the marginal probability functions of $\Delta\rho$ and $\Delta\theta$ are assumed to be gamma probability functions with two parameters where the respective shape functions are proportional to the considered time interval τ (more information on the use of gamma processes in maintenance optimization for structures can be found in Van Noortwijk[6]). The gamma process offers many benefits in terms of degradation modeling. As we will see in the remainder of this document, the self-explanatory parameters of the process allow us to associate to them physical meanings, making the integration of a maintenance action easier. Furthermore, the monotonous characteristics of the process is adequate to many pathologies found in civil engineering (such as creep, wear, and fatigue cracking)

Furthermore, the corrosion current density has an effect on the propagation of cracks, and vice-versa (mutual dependencies). This correlation is modeled in terms of mutual acceleration effects directly in each of the shape functions of the gamma distributions.

Finally, to simulate, we first seek to characterize the evolution in terms of the causal process (corrosion current density, equation (2)), then doing so for the respective effect process (crack width, equation (3)). We can then write $\forall(\rho, \theta) > 0$

$$\Delta\theta(\tau; \rho, \theta) \sim g(\alpha_\theta(\rho, \theta).\tau, \beta_\theta) \tag{1}$$

$$\Delta\rho(\tau; \rho, \theta, \Delta\theta) \sim g\left(\alpha_\rho(\rho, \theta, \Delta\theta).\tau, \beta_\rho\right) \tag{2}$$

Note that to simplify the identification process, we consider that the state dependence is exclusive to the shape functions: the scale functions β_θ and β_ρ are considered time independent. Therefore, we have to define the shape functions α_θ and α_ρ.

The choice of each shape function is motivated by the evolution of its respective indicator in time (Figure 1(a) and (b)). In other terms, the S-shaped condition state evolution of the corrosion current density (Figure 1(a) requires a bell-shaped shape function. The L-shaped condition state evolution of the crack width (Figure 1(b)) requires an akin shape function. As a result, we propose as shape functions $\forall(\rho, \theta) > 0$

$$\alpha_\theta(\rho, \theta) = (a_3.\rho + a_4).e^{\frac{-(\theta - a_1)^2}{a_2}} \tag{3}$$

$$\alpha_\rho(\rho, \theta, \Delta\theta) = \left(a_6.\left(\theta + \frac{\Delta\theta}{2}\right) + a_7\right).e^{-a_5.\rho} \tag{4}$$

Table 1. Definition of the parameters.

Parameter	Definition
β_θ, β_ρ	Proportionality factors common to different structures (materials)
a_1	Abscissa of the inflection point of the realizations of corrosion current density
a_2	Reflects the dispersion around the inflection point
a_3, a_6	Acceleration coefficients
a_4	Corrosion current density speed at the origin
a_5	Reflects the kinetics of the process ρ
a_7	Crack growth rate at the origin

The exponential parts of the shape functions govern the required shape of the shape function (e.g. bell shaped). The linear functions play an acceleration role, allowing by that to model the dependencies of the two processes.

One of the main motives in using degradation meta-models is to minimize the number of parameters, explaining the simple linear form of the acceleration function. However, if further knowledge on the correlation of the two physical indicators is available, it is possible to complexity these functions to account for the suitable acceleration and deceleration effects between the two indicators.

Now that the model has been defined, a physical meaning can be given for each parameter. In fact, the mathematical formulation of the model allows us to identify physical tendencies (or causalities) associated with each parameter. In Table 1, the physical meanings of each parameter are summarized.

Furthermore, Figures 2 and 3 illustrate the respective α_θ and α_ρ shape functions of the processes. Each shape function is state-dependent and is presented as a function of the two degradation indicators (ρ, θ).

In Figure 4, four trajectories are simulated to illustrate the model.

The following parameters are used for these illustrations

$$a_1 = 1,\ a_2 = 1,\ a_3 = 1,\ a_4 = 1.2,\ a_5 = 0.8,$$
$$a_6 = 1.8,\ a_7 = 2,\ \beta_\rho = 0.3 \text{ and } \beta_\theta = 0.3.$$

Estimation procedure

We assume the existence of a database formed by successive perfect measurements of the crack width and the rate of corrosion of n structures denoted $\{(\rho_t^{(j)}, \theta_t^{(j)}),\ t \geqslant 0, j \in 1, n\}$.

We propose here to estimate the parameters of the meta-model by the method of maximum likelihood on the existed database. However, due to the mathematical expression of the shape function, the estimation of the nine parameters leads to numerical instability when using conventional optimization procedures. To work around this problem, we have built a heuristic

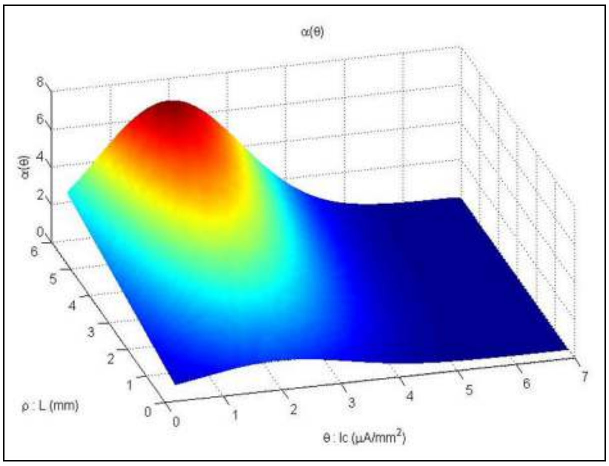

Figure 2. Bell-shaped shape function of the θ process—$\alpha_\theta(\rho,\theta)$.

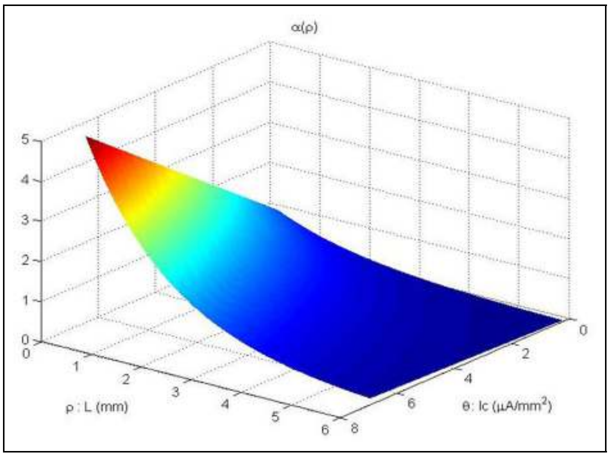

Figure 3. L-shaped shape function of the ρ process for constant $\Delta\theta$—$\alpha_\rho(\rho,\theta,\Delta\theta)$.

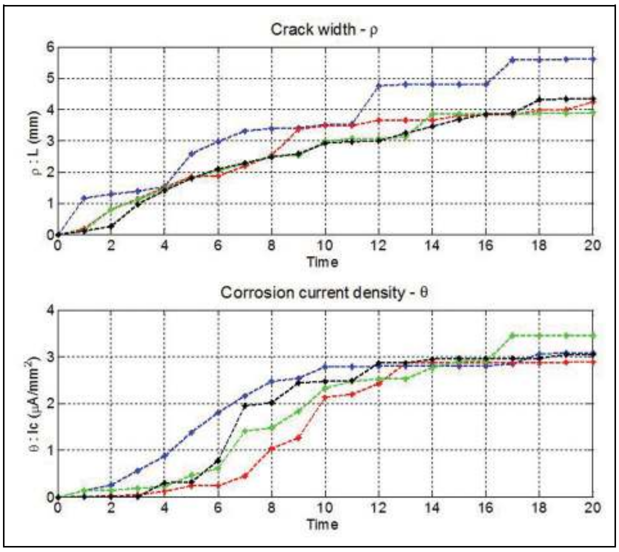

Figure 4. Four simulations of the bivariate model (represented using four colors).

based on the fixed-point theorem. This heuristic is applied iteratively to provide estimates of the parameters of the respective models equations (1) and (2).

Heuristic
Step 0.

1. Construction of the database $\{(\rho_t^{(j)},\theta_t^{(j)}),\ t\geqslant 0,\ j\in 1,n\}$(*resp*. $\{(\rho_t^{(j)},\theta_t^{(j)},\Delta\theta_t^{(j)}),\ t\geqslant 0,\ j\in 1,n\}$ with $\Delta\theta_t^{(j)}=\theta_{t+1}^{(j)}-\theta_t^{(j)}$)
2. Calculate the likelihood of equation (1) (*resp*.equation (2)) for the corresponding database
3. Initiate $\beta_\rho=\beta^{(0)}$ (*resp*.β_θ)

   Step j. While $\left|\hat{\beta}^{(j)}-\hat{\beta}^{(j-1)}\right|>\varepsilon$, do
4. Determinate the MLE estimates $\hat{a}_i{}^{(j)}$ for $\beta_\rho=\hat{\beta}^{(j-1)}$ (*resp*.β_θ)
5. Evaluate $\hat{\beta}^{(j)}$ as the estimator of the MLE for the just considered $\hat{a}_i{}^{(j)}$.

where MLE is the classical maximum likelihood estimation (MLE) method applied to each process separately. It may also be noted, databases in civil engineering might suffer from incompleteness. In such cases, this algorithm needs to be extend in order to impute the missing or erroneous information and to estimate the parameters of the model. The Stochastic Estimation Maximization algorithm is to be used.[17]

NB: we will not demonstrate the convergence of this fixed-point type algorithm. However, the large number of numerical experiments that we describe below portends the good properties of this algorithm.

In the next section, we propose to investigate the statistical performances of the proposed degradation meta-model in heterogeneous databases. The general idea is to question the potential use of heterogeneous databases to ameliorate the estimation process of the model's parameters when the original database is poor.

Statistical performance analyses

In El Hajj et al.,[10] the convergence of the estimation procedure is discussed through numerical analyses. These numerical analyses are conducted by simulating a database from defined parameters, and then the estimation's performance is assessed by means of the estimated average mean squared errors (MSE) on the estimated parameters.

The main result is illustrated by the paradigm in the inspections of a limited number of structures on their complete lifetime and the inspections of a higher number of structures inspected through the first years of their respective life. Because of the very long lifetime, it is concluded that such approach gives a real benefit in terms of applicability in civil engineering. Nevertheless, the classical problem of the non-homogeneity of a population in the database can degrade this conclusion.

We propose in the next subsection to analyze the adaptability of the meta-model toward the problem of non-homogeneity in a database, first, from a parameter inference point of view and then on a reliability metric consideration, the mean lifetime of a structure.

Robustness of the estimation procedure in respect to the heterogeneity of the database

Degradation parameters (cracking initiation and crack growth rate) are related to specific properties inherent to the studied structure. In civil engineering, it is clear that it can be difficult to qualify database samples as homogeneous because of the strong non-homogeneity of material, formulas and conception processes, the different in-service conditions, environment, or more previous maintenance actions. The question of the homogeneity in a database results in the selection of samples with very small sizes; this phenomenon is also amplified with the poorness of current database and the quality of the inspection policy.[18] As a consequence, the quality of the estimation process can be strongly degraded due to the lack of data.

In this study, we want to investigate the robustness of the proposed estimation algorithm face to non-homogeneity in a database. For this, we propose to generate 500 simulated databases with N trajectories (from the crack initiation to the end of life for each of the N structures). In each database, the non-homogeneity is obtained by integrating some controlled random variability on the given parameters of the proposed bivariate meta-model. Hence, for $N = 20$, five of the trajectories are simulated with the original parameters (no variability), five more with 5% variability, five more with 10%, and the last five with 15%. And then, for $N = 15$, five homogeneous trajectories, five more with 5% variability, and five with 10%. For $N = 10$, we have five homogeneous trajectories and five trajectories with 5% variability. And finally, $N = 5$ is formed by five homogeneous trajectories.

The variability sketches the non-homogeneity of the database and the challenge is in improving the homogeneity of the database and is illustrated by the decreasing from $N = 20$ structures to $N = 5$ (perfectly homogeneous case) for the estimation of the model parameters $\{\hat{a}_i, i = 1, \ldots, 9\}$ by removing five structures each time.

We propose to analyze the benefit of including more structures, even non-homogeneous ones, in the estimation process using the MSE given by

$$MSE = trace\left(E\left[(\widehat{\Theta}^{(N)} - \Theta)(\widehat{\Theta}^{(N)} - \Theta)^T\right]\right) \tag{5}$$

where Θ is the vector of the given parameters, and $\hat{\Theta}^{(N)}$ the vector of the estimated parameters using N trajectories. Table 2 contains the mean of the MSE obtained for each of the 500 databases, $\overline{MSE}$ (third row). The second row, $\overline{MSE}_0^{(N)}$, is the mean values obtained for homogeneous databases (no variation in the parameters for the whole simulated database) with different number of structures. The last row is the relative error given by

$$\bar{e} = \frac{\overline{MSE} - \overline{MSE}_0^{(N)}}{\overline{MSE}_0^{(N)}} \tag{6}$$

In this example, the following parameters are used

$$a_1 = 1,\ a_2 = 2,\ a_3 = 2,\ a_4 = 0.8,\ a_5 = 0.6,$$
$$a_6 = 1,\ a_7 = 1.4,\ \beta_\rho = 0.3 \text{ and } \beta_\theta = 0.3.$$

In this numerical experiment, we point out the potential benefits of considering additional data issued from heterogeneous structures. In this example, the increase in the relative error when $N = 20$ suggests that the homogeneity level should be optimized to ensure the estimation quality.

Table 2. Impact of the non-homogeneity level of the sample on the estimation performance.

N	5	10	15	20
$\overline{MSE}_0^{(N)}$	4.979	2.179	1.504	1.110
$\overline{MSE}$	4.979	2.502	1.518	1.223
$\bar{e}$	0	0.148	0.009	0.012

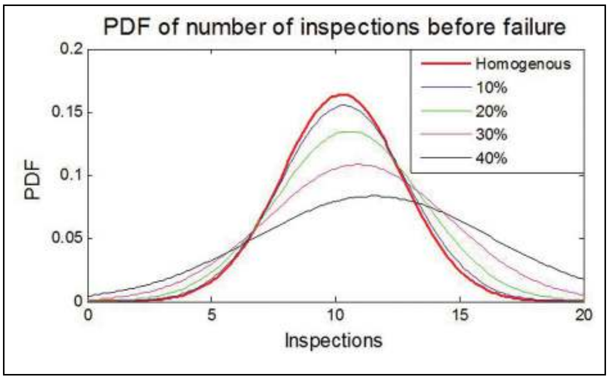

Figure 5. Probability distribution functions of the number of inspections before inspecting a failure.

Impact of the homogeneity level on the reliability performance

In this section, we illustrate the effect of variability on the parameters in terms of durability. A failure threshold on ρ is introduced and the lifetime is defined until the first inspection after the failure.

The non-homogeneity in the sample is directly modeled in terms of variability on the parameters. In Figure 5, probability distribution functions of the of the number of inspections before inspecting a failure are summarized for five cases where the variability sweeps the following values $0\%, \pm 10\%, \pm 20\%, \pm 30\%$ and $\pm 40\%$.

The results are obtained from 25,000 simulations, with the following parameters

Table 3. Impact of variability on the statistical moments of the number of inspections before failure.

	±0%	±10%	±20%	±30%	±40%
μ	10.26	10.34	10.56	10.91	11.53
e_μ(%)		0.82	2.94	6.37	12.48
σ	2.43	2.57	2.96	3.68	4.77
e_σ(%)	0	5.51	21.52	51.15	96.16

$$a_1 = 2,\ a_2 = 4.5,\ a_3 = 1.8,\ a_4 = 1.8,$$
$$a_5 = 0.65,\ a_6 = 1,\ a_7 = 1,\ \beta_\rho = 0.2 \text{ and } \beta_\theta = 0.2.$$

The non-homogeneity level could impact the durability estimation in two ways. First, some bias is introduced in the mean lifetime estimation when the variability on the parameters increases. For decision-making, this effect would lead to a bad inspection policy.

The second impact is related to the propagation of the uncertainty in the model, illustrated in the increase in the variance of the lifetime distribution. Table 3 sketches the evolution of the associated relative increase in the standard deviation.

In this numerical experiment, we can conclude that a homogeneity level in the 10% interval remains eligible for the estimation of the mean lifetime. Beyond this variability, the quantification of a structure in terms of durability is too hazardous.

Application to risk analysis

A structure is considered to be safe if the probability of failure P_f at any time is lower than a given threshold. For durability of concrete structures, Eurocode 2 requires to express the failure by comparing the crack width of concrete cover with a cracking threshold L. The later depends on both the characteristics of the structure and its environmental conditions.

In case of inspection, the decision criterion becomes the probability of having a failure before the next inspection. This probability is required to be lower than a threshold P_f. To illustrate the potential use of the model in risk-based analysis, we consider here a threshold $P_f = 0.05$.

The probability of a failure in the next inspection denoted $P_f(\rho_i, \theta_i)$ is a function of the current observation (ρ_i, θ_i) and given by the following equation

$$\begin{aligned} P_f(\rho_i, \theta_i) &= P(\Delta\rho + \rho_i > L \mid \rho = \rho_i, \theta = \theta_i) \\ &= \int_{L-\rho_i}^{+\infty} \int_{\theta_i}^{+\infty} g(x, y; \rho_i, \theta_i)\, dy\, dx \end{aligned} \tag{7}$$

For a selected range for $\rho (0 < \rho_i < L = 3\,\text{mm})$ and θ, simulations were done for estimating the probability of failure by Monte-Carlo method for every possible combination (ρ_i, θ_i) for a set of given parameters

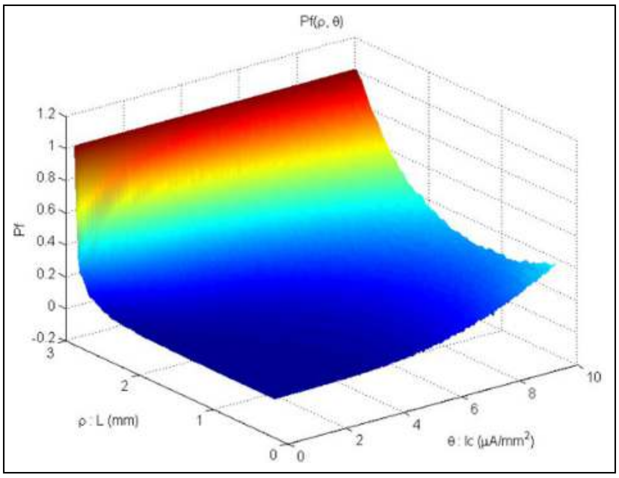

Figure 6. Probability of failure based on the degradation level (ρ_i, θ_i).

$$a_1 = 2,\ a_2 = 3,\ a_3 = 0.8\, a_4 = 0.8,\ a_5 = 0.5,$$
$$a_6 = 0.5, a_7 = 1.0,\ \beta_\rho = 0.3 \text{ and } \beta_\theta = 0.3.$$

The results are presented in Figure 6.

All simulations are carried out under MATLAB©. The use of this curve in reliability based management could be in a classical way where a P_f threshold defines an acceptance and critical areas. Therefore, we define an iso-curve as the line joining all observations (ρ_i, θ_i) having $P_f = 0.05$, and then we draw the iso-curve (green line) in Figure 7.

The iso-curve divides the plot in two areas: an acceptance reliability area where $P_f < 0.05$, and a critical reliability area where $P_f \geqslant 0.05$. The system is said to be safe for an observation (ρ_i, θ_i) in the acceptance reliability area (gray area) and unsafe in the critical reliability area. It is also easy to define a safety area with two thresholds and a specific attention or preventive action could be done to reduce the current risk level.

One major advantage of the proposed approach in reliability based management is that the decision can be modulated according to additional observation, given that θ can be seen as an acceleration factor of the cracking.

The aim of the following section is to investigate the effect of a potential error committed in the estimation of the parameters. Therefore, for the sake of this example, we consider a + 10% error on the parameters of

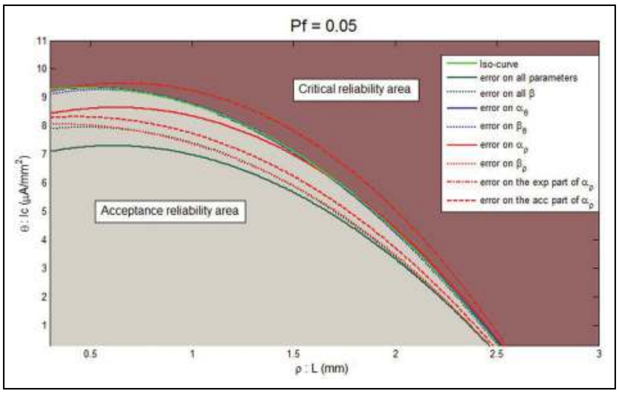

Figure 7. Fitted iso-curves of the degradation levels for a 0.05 probability of failure function of a + 10% error committed on several parameters of the meta-model and the simulations and mean of degradation levels.

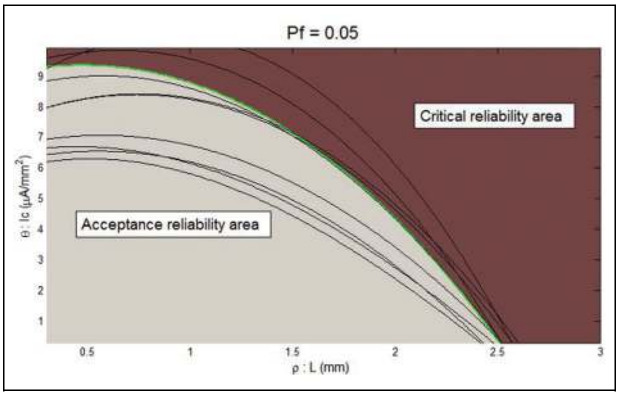

Figure 8. Estimated fitted iso-curves.

the bivariate model. The results are illustrated in Figure 7. In Figure 7, we see that a 10% error on the parameters of the θ process does not have a noticeable effect on the iso-curves, contrasting with an error committed on the parameters of the ρ process.

An error on the exponential part (a_5) of α_ρ (equation (4)) pushes the iso-curve upward, opposite to the error on the acceleration part (a_6, a_7) of α_ρ (equation (4)) which pushes the iso-curve downward. The parameter with the biggest impact on the iso-curve is β_ρ, followed by lesser impact from a_5, a_6, and a_7.

When 10% is added to the β_ρ parameter, the iso-curve is lowered, therefore triggering an early decision generating an over cost. However, a 10% error on the exponential part of α_θ pushes the iso-curve upward and therefore compromising on the "safety" of the decision.

When the iso-curve is lowered, an early decision is triggered causing additional costs. On the other hand, when the iso-curve is highbred, the safety of the decision is compromised by a late decision. However, the decision maker cannot distinguish between these two cases. Therefore, if the decision is based on this plot, an additional safety factor needs to be considered.

We take the case of $n = 10$ structures and $T = 20$ inspections where 10 realizations are simulated. Using the MLE algorithm, 10 sets of parameters for are estimated and the 10 corresponding iso-curves are then drawn in Figure 8.

In Figure 8, we witness the dispersion on both sides of the green iso-curve giving us an indecisive answer on whether we are over or under the no-error iso-curve. The safety factor is applied by lowering the estimated iso-curve by a distance equal to the range between the two furthest iso-curves. In Figure 8, the mean errors of the lowest and highest iso-curve are, respectively, 4% and 10% with a -17% and -4% error on the β_ρ. This safety factor will most probably generate an over cost without compromising on the security of the decision. In this study, we carried out a sensibility test and investigated the use of the bi-variate degradation meta model for risk-management applications. El Hajj et al.[17] further highlights the potential benefit of using a bi-variate approach for risk-management when compared to a mono-variate approach.

Up to now, maintenance actions were was not taken into account in the proposed degradation model. After a maintenance action, the structure's performance is modified. Therefore, it is necessary to update the degradation model to integrate these modification. We propose in the next section to describe how a maintenance action can be modeled within the proposed framework.

Maintenance action modeling

This section illustrates how a maintenance action is modeled in the meta-model. First, we will describe a maintenance action applied in the case of chloride-induced corrosion. Then we clarify the mathematical modeling of the maintenance action. A common maintenance action used in this case is the cathodic protection (CP).

CP

CP[19] is an electrochemical technique used to control the corrosion by making it the cathode of an electrochemical cell. CP systems protect metal reinforcement bars in concrete buildings and structures from corrosion and in some cases can prevent stress corrosion cracking.

It prevents corrosion by converting the active anodic sites on the reinforcement surface to passive cathodic sites by supplying electrical current or free electrons from an alternate source. It may be achieved by two ways depending on the supplied source of power: by the use of an impressed DC current from an electrical source or by the use galvanic action (also known as sacrificial anodes).

Galvanic action (or sacrificial anode). In the application of passive CP, galvanic (or sacrificial anode) is selected. The sacrificial anode is more electrochemically active (lower electrode potential) than the corroded reinforcement (cathode) and is electrically connected to the surface of the steel where it is exposed to an electrolyte.

The potential of the steel surface is then polarized until the surface has a uniform potential. At that stage, we are protecting the cathode. The corrosion is transferred from the reinforcement steel to the sacrificial anode, consuming material until eventually replaced.

The polarization of the reinforcement steel is done through migration of electrons from the anode to the cathode. Therefore, it is important that these two metals have a good electrically conductive contact. This type of CP is mostly used for local protection where we have a clear idea where the steel is under corrosion reaction.

Impressed current systems. Impressed current systems (ICS)[20] is generally an option where galvanic anodes fail economically or physically to deliver enough current in order to provide protection, for example, larger structures and higher electrolyte resistivity.

ICS is a set of anodes connected to a direct current (DC) power source. Sometimes, the DC is supplied by means of a transformer-rectifier connected to an AC powered by a supply, solar panels, wind power, or gas powered thermoelectric generators.

Then, the DC negative pole is connected to the reinforcement steel to be protected by ICS, and the positive is connected to the anodes. The output of the ICS is adjusted in a way to provide sufficient current to provide CP.

Effect of a maintenance action on the meta-model

Maintenance actions can have different effects on the process of corrosion and therefore on its parameters of interest. In our case, we do not remove concrete and a CP decelerates the corrosion.

We propose to model the effect of a CP action on the processes directly in the i_{corr} shape function by introducing a new parameter: m_1 which can be defined as the degradation acceleration factor after maintenance. Therefore, for the average corrosion current density, we multiply the θ process shape function by a constant m_1 (equation (8) and Figure 9), as a result

$$\alpha_\theta(\rho, \theta) = m_1 \times (a_3.\rho + a_4).e^{\frac{-(\theta - a_1)^2}{a_2}} \tag{8}$$

Maintenance techniques have been widely applied and studied, and their effect on the physical process are rigorously studied. So, the harder part in modeling the maintenance is to quantify m_1. The estimation process in case of maintenance action is beyond the scope of this article, but a MLE procedure similar to the one presented earlier can be used toward this aim using experimental data.

Let us consider for the sake of this example that a galvanic CP will slow the corrosion process by 10% and the impressed current CP by 20%; therefore, we will have, respectively, the following parameters for the maintenance parameters

- $m_1 = 0.75$ for the galvanic action and
- $m_1 = 0.5$ for the ICS.

For the purpose of illustrating the CP maintenance action, in Figure 10, we plot the mean simulations of the process showing how the CP slows the corrosion process as predicted.

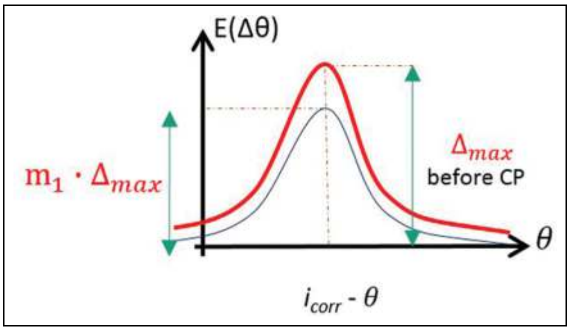

Figure 9. Mathematical modeling of a maintenance action on the shape functions.

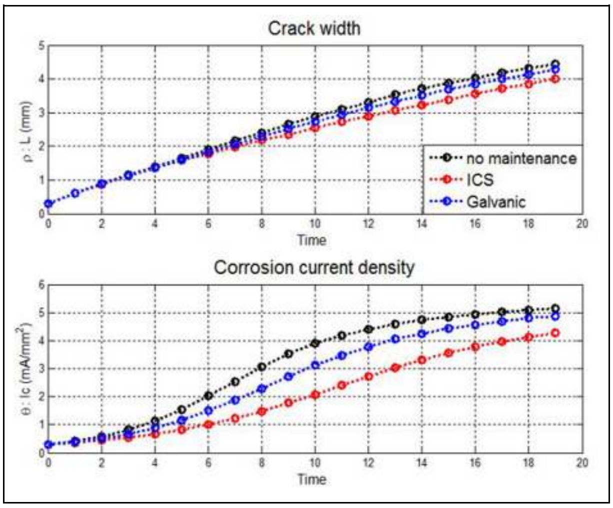

Figure 10. Mean simulations of the degradation indicators in case of CP maintenance actions.

Conclusion

In the first part of this article, the construction, mathematical formulation, and estimation algorithm of the data-driven bivariate meta-model were presented. Then, the meta-model's robustness was subjected to tests such as adaptability to non-homogeneous databases, applicability in a risk-based decision analysis, and maintenance actions modeling.

The performance of the model in a non-homogeneous context was evaluated. The results illustrated the ability of the meta-model's estimation process to respond to non-homogeneity in field-collected data to a certain level. Furthermore, from a decision-making perspective, having measurable indicators as outputs of the degradation meta-model allows to consider and recommend its use in risk-based contexts. This article illustrates the potential use of the proposed degradation meta-modeling approach for maintenance and risk management.

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