Corrections to: "On Lévy processes conditioned to stay positive"

Loïc Chaumont, R.-A. Doney

Electronic Journal of Probability

Vol. 13 (2008), Paper no. 1, pages 1–4.

Journal URL
http://www.math.washington.edu/~ejpecp/

Corrections to "On Lévy processes conditioned to stay positive"

L. Chaumont
LAREMA, Université d'Angers
2 boulevard Lavoisier
49045 Angers Cedex 01
E-mail: loic.chaumont@univ-angers.fr
http://math.univ-angers.fr/perso/~chaumont

R. A. Doney
Department of Mathematics
University of Manchester
Oxford Road, M13 9PL, Manchester
E-mail: rad@maths.manchester.ac.uk
http://www.maths.manchester.ac.uk/DeptWeb/Homepages/rad

Abstract

We correct two errors of omission in our paper, [2] .

Key words: Lévy process conditioned to stay positive, weak convergence, excursion measure.

AMS 2000 Subject Classification: Primary 60G51, 60G17.

Submitted to EJP on September 25, 2007, final version accepted December 18, 2007.

We would like to correct two errors of omission in our paper, [2]. The first occurs in equation (2.4), where we overlooked the possibility that the downgoing ladder time process has a positive drift. This happens if and only if 0 is not regular for $(0,\infty)$. If this drift is denoted by η, the correct version of (2.4) is

$$\begin{aligned} &\mathbb{P}_x(\tau_{(-\infty,0)} > \mathbf{e}/\varepsilon) = \mathbb{P}(\underline{X}_{\mathbf{e}/\varepsilon} \geq -x) \\ &= \mathbb{E}\left(\int_{[0,\infty)} e^{-\epsilon t} \mathbb{1}_{\{\underline{X}_t \geq -x\}}\, d\underline{L}_t\right) [\eta\varepsilon + \underline{n}(\mathbf{e}/\varepsilon < \zeta)], \end{aligned} \tag{1}$$

and the correct version of (2.5) is:

$$h(x) = \lim_{\varepsilon\to 0} \frac{\mathbb{P}_x(\tau_{(-\infty,0)} > \mathbf{e}/\varepsilon)}{\eta\varepsilon + \underline{n}(\mathbf{e}/\varepsilon < \zeta)}. \tag{2}$$

However this makes no essential difference to the proof of the following Lemma 1: we just need to replace $\underline{n}(\mathbf{e}_\varepsilon < \zeta)$ by $\eta\varepsilon + \underline{n}(\mathbf{e}_\varepsilon < \zeta)$ four times, and $\underline{n}(\zeta)$ by $\eta + \underline{n}(\zeta)$ in (2.6). The details can be seen in section 8.2 of [3]. We should also mention that (1) can be found in [1]: see equation (8), p 174.

The second omission is that we failed to give any proof of

Corollary 1. *Assume that 0 is regular upwards. For any $t > 0$ and for any $\mathcal{F}_t$ -measurable, continuous and bounded functional F,*

$$\underline{n}(F, t < \zeta) = k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}F).$$

The clear implication from our paper is that this follows immediately from our main result, Theorem 2, but this overlooks the singularity at zero of the function $1/h(x)$. Since this Corollary has been cited in a number of recent papers, we give here a full proof of it.

Proof. From (3.2) and Theorem 2 of [2] we see that, for any fixed $\delta > 0, t > 0$,

$$\underline{n}(F, t < \zeta, X_t > \delta) = k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}F, X_t > \delta),$$

and in particular, taking $F \equiv 1$,

$$\begin{aligned} \underline{n}(t < \zeta, X_t > \delta) &= k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}, X_t > \delta) \\ &= k \lim_{x\to 0} \mathbb{P}_x(X_t > \delta, \tau_{(-\infty,0)} > t)/h(x). \end{aligned}$$

Suppose we can show that

$$\underline{n}(t < \zeta) = k \lim_{x\to 0} \mathbb{P}_x(\tau_{(-\infty,0)} > t)/h(x). \tag{3}$$

Then, by subtraction,

$$\begin{aligned} \underline{n}(t < \zeta, X_t \leq \delta) &= k \lim_{x\to 0} \mathbb{P}_x(X_t \leq \delta, \tau_{(-\infty,0)} > t)/h(x) \\ &= k \lim_{x\to 0} \mathbb{E}_x^{\uparrow}(h(X_t)^{-1}, X_t \leq \delta). \end{aligned}$$

Since $\underline{n}(t<\zeta, X_t=0)=0$, if K is an upper bound for F, we also have

$$\lim_{\delta\to 0}\underline{n}(F, t<\zeta, X_t\le\delta)\le K\lim_{\delta\to 0}\underline{n}(t<\zeta, X_t\le\delta)=0,$$

and the required conclusion follows.

To prove (3) we start with (1), and, since we are assuming that 0 is regular upwards, the drift η in the downwards ladder time process is zero, so we can write it as

$$\int_0^\infty e^{-\varepsilon t}\mathbb{P}_x(\tau_{(-\infty,0)}>t)dt = h^{(\varepsilon)}(x)\int_0^\infty e^{-\varepsilon t}\underline{n}(\zeta>t)dt,$$

where $h^{(\varepsilon)}(x)=\mathbb{E}\left(\int_0^\infty e^{-\varepsilon t}\mathbb{I}_{\underline{X}_t\ge -x}dL_t\right)$. We know $0\le h^{(\varepsilon)}(x)\le h(x)$, so

$$\int_0^\infty e^{-\varepsilon t}\mathbb{P}_x(\tau_{(-\infty,0)}>t)dt \le h(x)\int_0^\infty e^{-\varepsilon t}\underline{n}(\zeta>t)dt.$$

But we also have

$$\begin{aligned}\liminf_{x\downarrow 0}\frac{\mathbb{P}_x(\tau_{(-\infty,0)}>t)}{h(x)} &\ge \lim_{\delta\downarrow 0}\lim_{x\downarrow 0}\frac{\mathbb{P}_x(\tau_{(-\infty,0)}>t, X_t>\delta)}{h(x)}\\ &= \lim_{\delta\downarrow 0}\underline{n}(\zeta>t, X_t>\delta)=\underline{n}(\zeta>t).\end{aligned}$$

Together, these prove that

$$\lim_{x\downarrow 0}\int_0^\infty e^{-\varepsilon t}\frac{\mathbb{P}_x(\tau_{(-\infty,0)}>t)dt}{h(x)} = \int_0^\infty e^{-\varepsilon t}\underline{n}(\zeta>t)dt.$$

Thus the measure with density $\mathbb{P}_x(\tau_{(-\infty,0)}>t)/h(x)$ converges weakly to the measure with the continuous density $\underline{n}(\zeta>t)$. But if $0<c<t$ are fixed we have

$$\begin{aligned}\lim_{x\to 0}\mathbb{P}_x(\tau_{(-\infty,0)} &> t)/h(x)\ge c^{-1}\lim_{x\to 0}\int_t^{t+c}\mathbb{P}_x(\tau_{(-\infty,0)}>s)ds/h(x)\\ &= c^{-1}\int_t^{t+c}\underline{n}(\zeta>s)ds\ge\underline{n}(\zeta>t+c),\\ \lim_{x\to 0}\mathbb{P}_x(\tau_{(-\infty,0)} &> t)/h(x)\le c^{-1}\lim_{x\to 0}\int_{t-c}^{t}\mathbb{P}_x(\tau_{(-\infty,0)}>s)ds/h(x)\\ &= c^{-1}\int_{t-c}^{t}\underline{n}(\zeta>s)ds\le\underline{n}(\zeta>t-c),\end{aligned}$$

and letting $c\downarrow 0$ we conclude that (3) holds, and hence the Corollary. □

Acknowledgement *We thank Erik Baurdoux and Mladen Savov for bringing these errors to our attention.*

References

[1] J. Bertoin. *Lévy Processes.* Cambridge University Press, Cambridge, (1996). MR1406564

[2] L. Chaumont and R. A. Doney. On Lévy processes conditioned to stay positive. Electronic J. Probab., **10,** 948-961, (2005). MR2164035

[3] R. A. Doney. *Fluctuation Theory for Lévy Processes.* Lecture Notes in Math. 1897, Springer, Berlin, (2007). MR2320889