Analytical Expressions of Diffraction'Free Beams Obtained by Diffraction on an Opaque Disk

Qiulin Huang, Sébastien Coëtmellec, Anne Louis, Fabrice Duval, Hervé Leblond, Marc Brunel

Analytical Expressions of Diffraction' Free Beams Obtained by Diffraction on an Opaque Disk

Q. Huang[1,2], S. Coëtmellec[1], A. Louis[3], F. Duval[3], H. Leblond[4], and M. Brunel[1]

[1]UMR 6614 CORIA CNRS, Université de Rouen, France
[2]Xidian University, Xi'an, China
[3]IRSEEM, ESIGELEC, Saint-Etienne du Rouvray, France
[4]LΦA EA 4464, Université d'Angers, France

Abstract— We establish approached analytical expressions that show that the beam produced after diffraction of a gaussian beam by an opaque disk and collimation by a lens can be described by a diffraction-free J_0 Bessel function. We further show that a similar analytical expression can be established in the case of femtosecond pulses.

1. INTRODUCTION

Diffraction-free beams have attracted much attention since pioneer work of Durnin and co-workers [1]. Different beam-shaping techniques exist, based on the use of an annular aperture, a computer generated-hologram or an axicon [2]. Although an annular aperture induces many losses, the technique based on this element has recovered much interest in view of recent studies involving sub-wavelength apertures [3]. They could indeed allow the design of systems in sub-wavelength optics.

We have recently proposed another method based on the occultation of an incident beam. We could indeed demonstrate that diffraction of a Gaussian beam by an opaque disk leads to the generation of a diverging Bessel'like beam, that can then be collimated with a lens into a diffraction-compensated beam [4, 5]. Numerical developments could corroborate experimental results. They showed that the beam diffracted by the opaque disk can be expressed as a sum of approximately 20 Bessel functions of odd orders. Unfortunately, these expressions are so complex that they do not allow to establish any simple expression of the diffraction-compensated beam after the collimating lens. Numerical integration is required to obtain correct simulations. We present in this communication a simplest formulation. After some simplifications that are detailed expressly, the beam is shown to be correctly described by a zeroth-order Bessel diffraction-free beam. We further show that this expression can be generalized to the case of 100 fs incident pulses.

2. DIFFRACTION BY AN OPAQUE DISK

We consider the diffraction of a Gaussian beam by an opaque disk, as described in Figure 1. A CW laser beam is first filtered through a pinhole. The filtered beam, which is a quasi-gaussian beam, is then focused with a lens L_1 An opaque disk whose diameter D is 300 µm is positioned just before the focus point. It is well centered on the optical axis, but behind the focus point. A lens L_2 collimates the beam diffracted by the opaque disk. The opaque disk is exactly positioned at the focus point of the lens L_2. The diffracted pattern is observed with a CCD camera.

According to the paraxial approximation, the expression of the scalar electric field diffracted by the opaque disk is given at distance z after the disk by:

$$u(x,y,z) = -\iint_{disk\ screen} \frac{2\,i\,k}{4\pi(z-z_q)} e^{-\frac{x_q^2+y_q^2}{w(z_q)^2}} e^{-ik\frac{x_q^2+y_q^2}{2R(z_q)}} e^{\left(ik(z-z_q)+\frac{(x-x_q)^2+(y-y_q)^2}{2(z-z_q)}\right)} dx_q\, dy_q$$

$w(z_q)$ and $R(z_q)$ represent the waist and the beam radius of the incident Gaussian beam, in the plane of the opaque disk. After some developments, this integral can be written

$$u(x,y,z) = -\frac{2ikD^2 e^{ik(z-z_q)} e^{-a(z)}}{8(z-z_q)} e^{i\frac{k\rho^2}{2(z-z_q)}} \sum_{m=0}^{+\infty} (-1)^m \left(\frac{1}{2a(z)}\right)^{m+1} \left(\frac{k\rho D}{2(z-z_q)}\right)^m J_m\left(\frac{k\rho D}{2(z-z_q)}\right) \quad (1)$$

with: $a(z) = \frac{D^2}{4}\left(\frac{1}{w(z_q)^2} - ik\left(\frac{1}{2(z-z_q)} - \frac{1}{2\,R(z_q)}\right)\right)$.

Figure 2 shows the comparison between an experimental profile, the theoretical development of reference [4] and the present expansion considering only the coefficient $m = 0$ (first-order approximation). For these experimental results, the laser source is a CW He-Ne laser emitting at

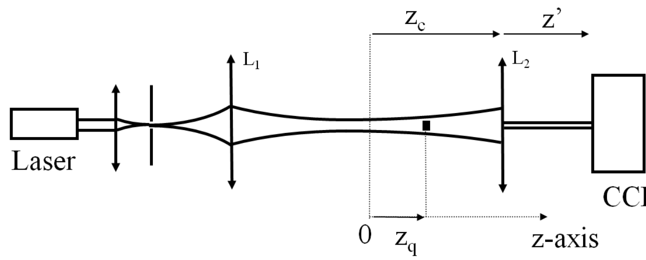

Figure 1: Experimental set-up.

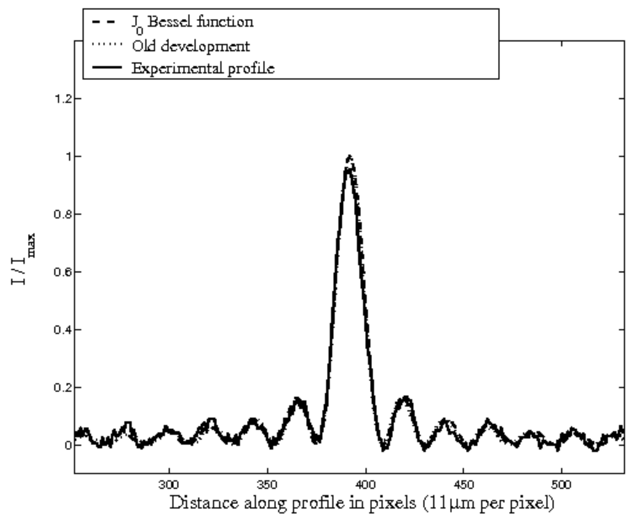

Figure 2: Comparison between the experimental profile, the new development limited to the sole J_0 Bessel function, and the old development of reference [4] which required the sum of 20 Bessel functions of odd orders.

632.8 nm. The focal length of L_1 is 10 cm. Focus point is 15 cm after the lens L_1. The diameter of the focus point is 70 µm at $1/e$. The opaque disk is 3.5 mm behind the focus point. The diffracted pattern is observed with a CCD camera positioned 10.5 cm after the opaque disk (Lens L_2 is not present in this first experiment). We can see that the first-order approximation using only a J_0 Bessel function allows a good fit of experimental results. Expression (1) can thus be reasonably simplified by the zeroth-order Bessel function in paraxial approximation. So, we get

$$u(x,y,z) = -\frac{2ikD^2 e^{ik(z-z_q)}}{8(z-z_q)} e^{i\frac{k\rho^2}{2(z-z_q)}} \frac{e^{-a(z)}}{2a(z)} J_0\left(\frac{k\rho D}{2(z-z_q)}\right) \tag{2}$$

with $\rho = \sqrt{x^2+y^2}$.

Note that the expansion limited to the sole J_0 Bessel function does never diverge. For comparison, expansions of higher orders would be more precise in their domain of convergence, but they would be limited to a domain of convergence given by $\rho < 4|a|(z-z_q)/(kD)$. In this case, the limit is $\rho < 2.5$ mm.

3. DIFFRACTION FREE BEAMS AFTER COLLIMATION BY A LENS

let us now consider collimation by the lens as detailed in Figure 1. On the plane $z = z_c$ where the collimating lens is located, the scalar field is reduced to the zeroth-order Bessel function in paraxial approximation. Propagation through the lens is then described by a transmitting phase factor, while propagation in free space after the lens can be expressed with a Fresnel transform. Assuming that the radius R of the lens L_2 is very large, these operations lead to a very simple result. If the opaque disk is located just on the focus point of the lens L_2, that is $f_{\{L2\}} = z_c - z_q$, The amplitude of the scalar field on the plane z' after the collimating lens L_2 can indeed be written:

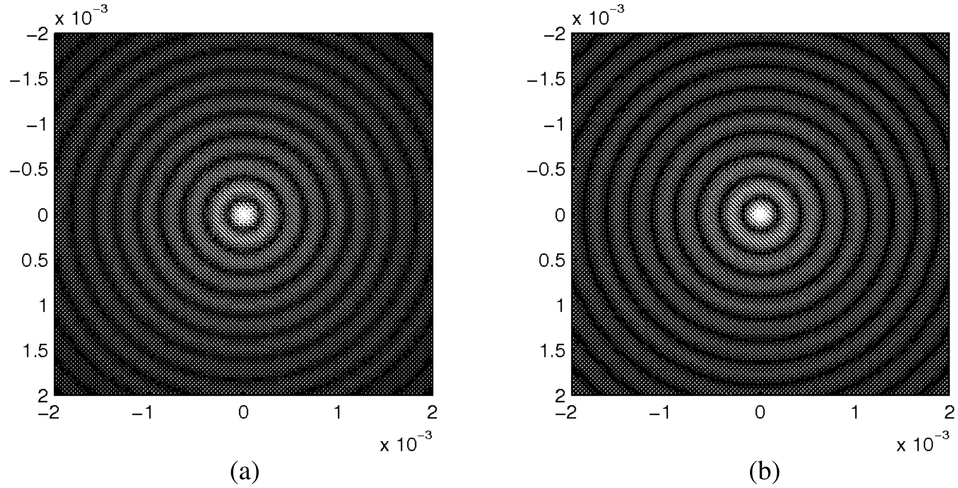

Figure 3: Comparison between (a) the old development of reference [4] which required the sum of 20 Bessel functions of odd orders and (b) the new development limited to the sole J_0 Bessel function.

$$u(x', y', z') = -\frac{2ikD^2 e^{ik(z'-z_q)} e^{-a(z_c)}}{16a(z_c)(z_c - z_q)} e^{-i\frac{k(z'-z_c)D^2}{8(z_c-z_q)^2}} J_0\left(\frac{k\rho' D}{2(z_c - z_q)}\right) \text{ with } \rho' = ({x'}^2 + {y'}^2)^{1/2''} \quad (3)$$

To illustrate this, Figure 3 shows a comparison between the beam patterns predicted 30 cm after the collimating lens using the old expansion of reference [4] over 20 Bessel functions of odd orders (Figure (a)), and the pattern predicted using the first-order J_0 Bessel function approximation (Figure (b)). Other parameters are the opaque disk diameter $D = 300\,\mu\text{m}$, $z_q = 3.5\,\text{mm}$, and $f_{\{L2\}} = 25\,\text{mm}$. Although they are not reported here, transverse profiles show a good quantitative accordance between experimental profiles and both theoretical developments. The J_0 Bessel approximation (3) is thus a good approached expression of the nondiffracting beam and is theoretically justified.

4. FEMTOSECOND DIFFRACTION FREE PULSES

Femtosecond pulses exhibit a large spectrum and the previous model cannot be used anymore. We consider ultrashort pulses with constant waist width under the paraxial approximation. Their propagation can be described by a combination of Fresnel diffraction for each spectral component, and a temporal filter for proper superposition of the components [6]. In the temporal Fourier domain, we can write $\tilde{E}(\vec{r}, \omega) = \tilde{U}(\vec{r}, \omega)\tilde{G}(\omega)$ where $\tilde{E}(\vec{r}, \omega)$ is the Fourier transform of the diffracted field $E(\vec{r}, t)$, and $\tilde{G}(\omega) = \exp(-(\omega - \omega_0)^2/\Delta\omega^2)$ is the spectrum of the pulses. By using the Parseval theorem, we obtain the diffraction intensity:

$$I(\rho, z) = \frac{C}{16(z - z_q)^2} e^{-\frac{D^2}{2w(z_q)^2}} \int_{-\infty}^{+\infty} \left|\tilde{U}(\vec{r}, \omega)\right|^2 \left|\tilde{G}(\omega)\right|^2 d\omega$$

where C is a normalization constant. It gives:

$$I(\rho, z) = \frac{Cw(z_q)^4}{16(z - z_q)^2} e^{-\frac{D^2}{2w(z_q)^2}} \int_{-\infty}^{+\infty} \frac{1}{1 + \tau^2\omega^2} e^{-\left(\frac{\omega-\omega_0}{\Delta\omega}\right)^2} J_0^2(\alpha\omega) d\omega$$

with $\alpha = \rho D/(2c(z - z_q))$ and $\tau = w(z_q)^2/(2c)((z - z_q)^{-1} - R(z_q)^{-1})$. Typical values of the parameters are $\alpha \approx 10^{-14}\,\text{s}$, $\tau \approx 10^{-16}\,\text{s}$, $\omega_0 \approx 3000\,\text{THz}$, $\Delta\omega \approx 30\,\text{THz}$. Introducing the new variable $x = (\omega - \omega_0)/\Delta\omega$, it is thus possible to expand the integral of previous relation in a series of powers of $\Delta\omega$. Simulations are presented in Figure 4. They show simulations using first-order, third-order and fifth-order Taylor–type expressions and the old integration of reference [6] in the case of 100 fs pulses. All simulations deliver very similar results in our case, because the spectrum width is relatively small. It appears that the first-order expansion is sufficient. Assuming then that the lens radius is very large, the expression of the intensity after the collimating lens can be evaluated and after some steps, it can be approached by:

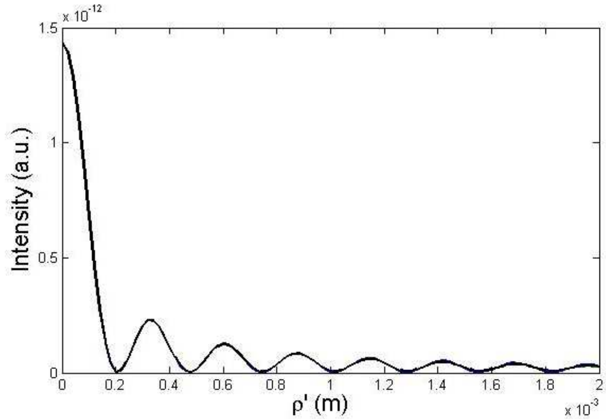

Figure 4: Comparison between first-order, third-order and fifth-order Taylor-type expressions and old numerical treatment of reference [6]. All curves are superposed.

$$I(\rho', z') = \frac{Cw(z_q)^4}{16(z_c - z_q)^2} e^{-\frac{D^2}{2w(z_q)^2}} \frac{\Delta\omega\sqrt{\pi} J_0^2(\beta\omega)}{1 + \tau^2\omega_0^2} \quad \text{with} \quad \beta = \rho' D/(2c(z_c - z_q)).$$

This relation shows the diffraction free nature of the beam. This expression represents a very important simplification of previous procedures which required long numerical developments as detailed in reference [6].

5. CONCLUSION

In conclusion, we have established approached analytical expressions that show that the beam produced after diffraction of a gaussian beam by an opaque disk and collimation by a lens can be described by a diffraction-free J_0 Bessel function. We have further showed that a similar analytical expression can be established in the case of femtosecond pulses. Those simple analytical relations allow the design of systems in different wavelength regions, particularly in the microwave regime, while very long computing times were necessary using previous numerical developments.

ACKNOWLEDGMENT

This work is supported by Carnot ESP.

REFERENCES

1. Durnin, J., J. J. Miceli, and J. H. Eberly, "Diffraction free beams," *Phys. Rev. Lett.*, Vol. 58, 1499–1501, 1987.
2. McGloin, D. and K. Dholakia, "Bessel beams: Diffraction in a new light," *Contemp. Phys.*, Vol. 46, 15–28, 2005.
3. Li, Z., K. Boratay Alici, H. Caglayan, and E. Ozbay, "Generation of an axially asymmetric Bessel-Like beam from a metallic subwavelength aperture," *Phys. Rev. Lett.*, Vol. 102, 143901, 2009.
4. Brunel, M. and S. Coëtmellec, "Generation of non diffracting beam through an opaque disk," *J. Opt. Soc. Am. A*, Vol. 24, 3753–3761, 2007.
5. Brunel, M., D. Mgharaz, and S. Coëtmellec, "Generation of diffraction-compensated beams with a pigtailed laser diode," *IEEE Photon. Technol. Lett.*, Vol. 20, 742–744, 2008.
6. Brunel, M., D. Mgharaz, and S. Coëtmellec, "Generation of femtosecond diffraction-compensated beam through an opaque disk," *Opt. Express*, Vol. 16, No. 14, 10390–10397, 2008.